 LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

**M.Sc.** DEGREE EXAMINATION - **MATHEMATICS**

SECOND SEMESTER – **APRIL 2012**

# MT 2812 - PARTIAL DIFFERENTIAL EQUATIONS

Date : 21-04-2012 Dept. No. Max. : 100 Marks

Time : 9:00 - 12:00

Answer all questions. Each question carries 20 marks.

1. (a) Using Charpit’s method solve . (5)

(OR)

1. Solve. (5)
2. Obtain the condition for compatibility of f(x, y, z, p, q) = 0 and g(x, y, z, p, q) = 0.
3. Show that, are compatible and find its solution. (7 + 8)

(OR)

1. Determine the characteristics of z = p2 – q2 and find the integral surface which passes through the parabola 4z + x2 = 0, y = 0. (15)
2. (a) If f and g are arbitrary functions show that is a solution of provided . (5)

(OR)

(b) Solve . (5)

(c) Define affine transformation and prove that the sign of the discriminate of a second of second order partial differential equation is invariant under the general affine transformation. (15)

(OR)

(d) Obtain the canonical form of the parabolic partial differential equation.

(e) Reduce to canonical form. (10 +5)

3. (a) Obtain the Poisson’s equation. (5)

(OR)

(b) D’Alembert’s solution of the one-dimensional wave equation. (5)

(c) Solve a two dimensional Laplace equation subject to the boundary conditions; u(x, 0), u (x, a) = 0, u (x, y) → 0 as x → ∞, where x ≥ 0 and 0 ≤ y ≤ a.

(15)

(OR)

(d) State and prove Interior Dirichlet Problem for a Circle. (15)

4. (a) Solve the wave equation given by , , subject to the initial conditions , , . (5)

(OR)

(b) Find the steady state temperature distribution *u*(*x*, *y*) in a long square bar of side π with one face maintained at constant temperature *u*0 and the other faces at zero temperature. (5)

(c) Use Laplace transform method, to solve the initial value problem

0 < *t* < ∞ subject to the conditions *u*(0, *t*) = 0, *u*(*l*, *t*) = *g*(*t*), 0 < *t* < ∞ and *u*(*x*, 0) = 0, 0 < *x* < *l*. (15)

(OR)

(d) State and prove Helmholtz Theorem. (15)

5. (a) Using Fredholm determinants, find the resolvent kernel when *K*(*x*, *t*) = *xet*, *a* = 0,

*b* = 1. (5)

(OR)

(b) If a kernel is symmetric then prove that all its iterated kernels are also symmetric.

(5)

(c) Find the solution of Volterra’s integral equation of second kind by the method of successive substitutions. (15)

(OR)

(d) State and prove Hilbert theorem. (15)

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